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A NEW FORM OF DIRECT-READING CANDLEPOWER SCALE AND RECORDING DEVICE FOR PRECISION PHOTOMETERS

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CONTENTS

I. INTRODUCTION

The new form of candlepower scale herein described was designed for use in the precision photometric work of the Bureau of Standards, and is the result of an attempt to combine the direct-reading feature of the ordinary commercial photometer candlepower scale with the accuracy possible in the use of the precision photometer. It is intended to be used in connection with an automatic recording device, also designed for use in the Bureau of Standards, and described in the transactions of A. I. E. E.[1] The automatic recorder eliminates the labor and possible errors of reading and recording the great number of settings made in a series of photometric measurements, while the direct-reading scale described below eliminates the labor and possibilities of error involved in the considerable computations heretofore necessary in finding the values of the lamps in terms of the mean of a group of standards.

II. THE PHOTOMETRIC APPARATUS AND THE DETERMINATION OF CANDLEPOWER

1. THE PRECISION PHOTOMETER AND METHOD OF MEASUREMENT

The precision work of the bureau in the photometry of electric incandescent lamps is done on a standard photometer of the Reichsanstalt pattern, supplied with a Lummer-Brodhun contrast screen, and the measurements are made by the substitution method; that is, the standards and the lamps to be compared with them are placed, in turn, in the socket near the left end of the bar, or test side, and are balanced against a comparison lamp placed near the right end, or comparison side. In order to eliminate all variables on the comparison side, and thus carry out the substitution method more perfectly, the two separate carriages supporting the comparison lamp and the photometer screen, respectively, are connected rigidly by means of adjustable rods (which are set before measurements are made), in order to maintain a constant distance between the comparison lamp and the screen, this distance being so determined as to throw the balance

[1] In a paper on "Carbon Filament Lamps as Photometric Standards," presented by E. B. Rosa and G. W. Middlekauff at the Twenty-seventh Annual Convention, June 28–July 1, 1910.

for a lamp of given candlepower at or near some desired distance from the lamp. The illumination of the comparison side of the screen is thus kept constant throughout the set of measurements, and the candlepowers of the lamps compared are directly proportional to the squares of their respective distances from the screen when it is set at photometric balance.

2. OLD METHOD OF READING AND RECORDING SETTINGS

Before the introduction of this new device the photometer settings were read by the observer from a centimeter scale graduated on the bar, and the readings were recorded by hand in a book provided for the purpose. From five to fifteen settings and corresponding readings of the scale, according to their range, were made on each lamp, thus making it necessary for the observer to record from two to even as many as five times during the measurement of each lamp, it being impossible to retain in mind more than about three readings at a time for correct recording.

This method of reading and recording settings is objectionable, because it consumes considerable time and involves the liability of error, both in reading and in recording, besides unnecessarily fatiguing the eye by alternate reading of photometer screen and scale.

3. OLD METHOD OF COMPUTING CANDLEPOWER

The readings having been recorded and the mean reading found for each lamp, it was then necessary, in order to find the candlepower values, to perform a tedious set of computations as shown by the following formula:

$$X=\frac{A+B+C+D+E+F}{a^2+b^2+c^2+d^2+e^2+f^2}.x^2$$

where A, B, C, D, E, and F are the values of the six standards (it being desirable to use at least this number in every set of measurements) and X is the value of any lamp compared with them; while a, b, c, d, e, f, and x are the corresponding, respective, mean distances of the lamps as read from the centimeter scale on the bar. Although these computations were much abbreviated by the use of a set of tables prepared especially for this purpose, nevertheless the work was very tedious, involving, at the same time, the

liability of error, thus making it absolutely necessary to have all the numerical work checked by a second computer whose task was equally laborious and required fully as much time as that of the first.

The ordinary direct-reading candlepower scale, such as used on commercial photometers, can not well be adapted to this class of work, because for precise measurement it is practically impossible to adjust the comparison lamp properly to "fit" the scale with the necessary accuracy; especially since, as stated above, it is desirable to employ at least six standards in the adjustment.

III. THE NEW APPARATUS

1. THE NEW FORM OF CANDLEPOWER SCALE

It occurred to the author that if the usual process of adjusting the comparison lamp to fit the candlepower scale were reversed and the scale were so constructed that it would adapt itself to the adjustment of the comparison lamp within certain reasonable limits the difficulty in using a commercial photometer scale for precision work could be avoided and the computations for candlepower eliminated. It was found that such a scale could be computed and constructed in a very simple and satisfactory manner, as explained below and as will be understood by referring to the accompanying Fig. 2.

The scale proper is the rectangular area *ABDC* and is calculated on the basis that a 16-cp lamp is to be measured with the photometer screen at a distance of approximately 120 cm from the lamp, a range of 6 cm either side of 120 cm being allowed in the construction of the scale. The sides *AB* and *CD* of the rectangle are two ordinary linear candlepower scales calculated to suit the arrangement of the photometer as described in the second paragraph above for two different adjustments of the comparison lamp, and the points of graduation were determined by means of the formula

$$\frac{x^2}{a^2}=\frac{X}{16} \quad \text{or} \quad x=\frac{a}{4}\sqrt{X}$$

where a is the distance of the 16-cp point of graduation from the lamp (or zero of the scale), being 126 cm for the scale *AB* and 114

cm for the scale CD, and x is the corresponding distance of any other cp point X on the scale that is being calculated. The rectangle was made 30 cm in length so as to include the candlepower points most commonly required in standardizing work, these being indicated along the sides AB and CD. The balance

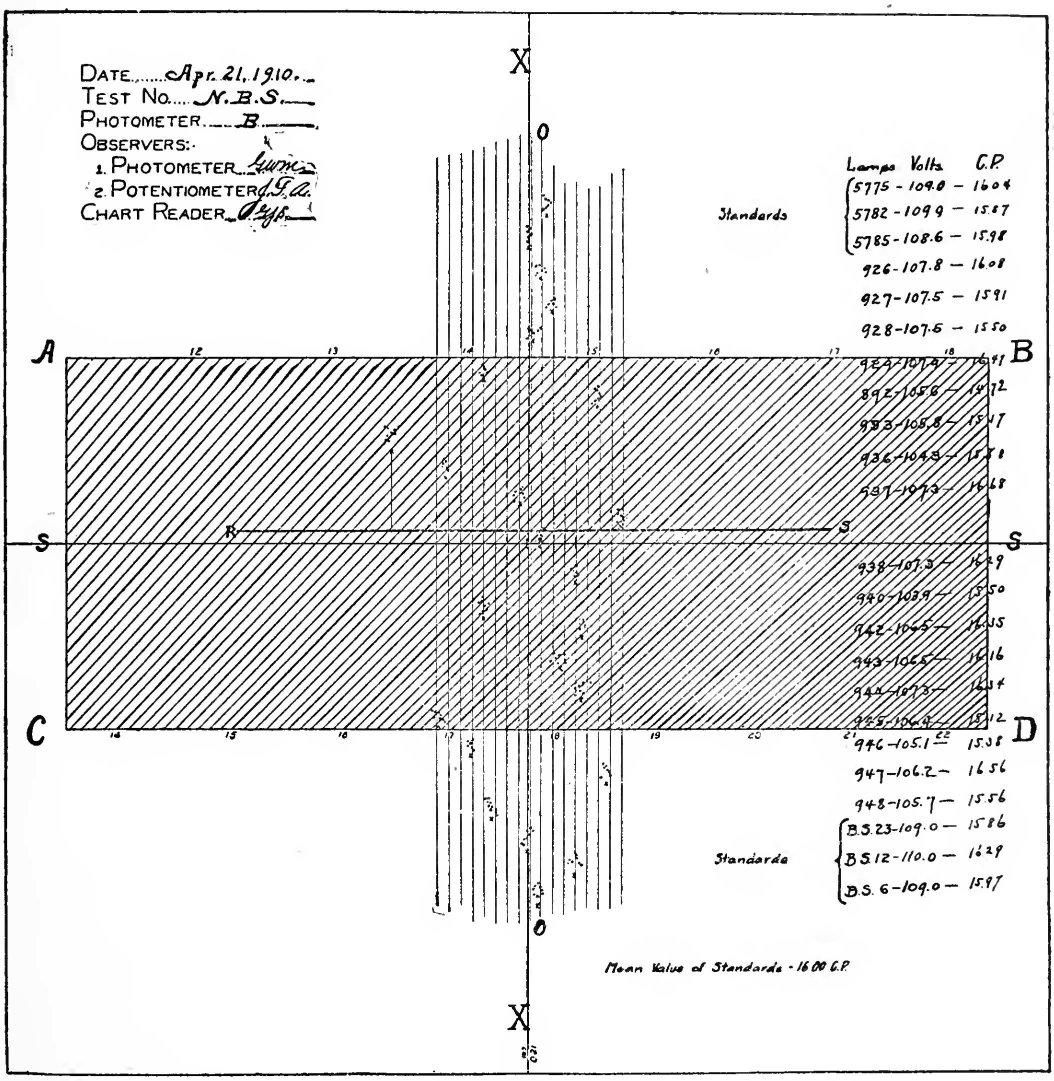


Fig. 2.—*Candlepower Scale and Record Chart. Comparison Lamp Movable with Photometer Screen*

for lamps having values above or below those indicated, for instance, 32-cp or 8-cp lamps, may be thrown within this range by means of rotating sectored disks placed on the proper side of the photometer screen or, in the case of low-intensity lamps, by a method described later. The diagonal lines, all of which are

straight, join the corresponding one-tenth candlepower points of graduation in the two scales; and since these two scales are divided proportionately, all the diagonal lines must converge toward a common point in a line drawn through zero and perpendicular to both scales. The slope of this system of lines depends upon the distance between *AB* and *CD*, which may be chosen arbitrarily. In order to prevent too much crowding of the diagonal lines on the one hand and too large a rectangle on the other, the 16-cp line was drawn so as to make an angle of 45° with the sides *AB* and *CD*, the other lines of the system being drawn in their proper directions accordingly. The rectangle thus becomes 12 cm in width and, as stated above, it was arbitrarily made 30 cm in length.

Now, any line drawn across the diagonal lines and parallel to the length of the rectangle will be divided in exactly the same proportion as *AB* and *CD* and will, therefore, be a candlepower scale also. Hence this rectangle includes an indefinite number of linear candlepower scales and corresponding to any fixed adjustment of the comparison lamp that will throw the 16-cp balance at any point within the 12-cm range between the limits of the 16-cp diagonal line (*i. e.*, between 114 cm and 126 cm from the lamp), there will be, in the rectangle, a linear scale that will exactly fit the comparison lamp adjustment. The method of determining the exact position of the proper linear scale for any given adjustment will be fully explained below.

Two important reference lines *XX* and *SS* are drawn across the figure perpendicular to each other and to the sides of the rectangle, intersecting the 16-cp diagonal line at its middle point, which is also the middle point of the rectangle and is at exactly 120 cm from the lamp located at zero of the scale. Hereafter, the line *XX* will be called the "index line," being the line by which the diagonal scale is set at its proper distance from the lamp mounted at zero on the photometer bar, and the line *SS* will be called the "standard scale," as it is the linear scale on which 16 cp falls at exactly 120 cm from the lamp, this being the distance adopted in the bureau's practice as standard in the comparison of electric incandescent lamps of 16-cp intensity, the illumination of the photometer screen under these conditions being approximately one foot-candle.

2. THE PAPER RECORD SHEET

The rectangular figure, as described above, was very accurately drawn just double the required dimensions, and after being photographed down to the correct size, it was transferred to a copperplate etching from which it was printed, in olive green, on sheets of white paper about 35 cm square, one of these sheets being intended for each separate set of lamps photometered.

The paper used for this purpose was specially prepared to better withstand changes in dimension due to ordinary variations in atmospheric humidity. As a further precaution, the longer dimension of the rectangle, which is the important one to keep constant, was printed parallel to the machine direction of the paper; that is, to the direction in which it is the least affected by changes in humidity.

By actual test made at the bureau on several samples of the paper about 35 cm square, it was found that under an increase in humidity from "dry" to saturated, the average total increase in dimension in the machine direction of the paper was about 0.9 mm, while at right angles to this direction the average total increase was 5.7 mm.

Now, any approximately uniform change in the width of the rectangle requires no consideration, because it has no effect upon the accuracy of the scale. It simply changes the slope of the lines, as a system, without altering the proportion in which they divide any line drawn across them and parallel to the length of the rectangle. A change of 0.9 mm in the length of the rectangle, however, means a change of 0.45 mm in the relative position of a point at the end of the scale with reference to the index line which is always placed at its proper distance from the lamp photometered; and 0.45 mm change in length is equivalent to about 0.01 candle, or 0.08 of 1 per cent of 12 cp at the lower end of the scale, and to about 0.015 candle, or 0.07 of 1 per cent of 22 cp at the upper end. Therefore, the maximum error possible in candlepower readings under the most unfavorable conditions can not be as much as 0.1 of 1 per cent. Indeed, numerous measurements of the length of the scale at various times and under various degrees of atmospheric humidity have failed to show any variation large enough to be detected by means of a millimeter scale. Hence with

the candlepower scale printed parallel to the machine direction, on paper of this kind, no appreciable error is introduced in candlepower readings as a result of any possible variations in atmospheric humidity, and especially is this true if all the settings are made, as they should be, as near the central portion of the paper as possible.

3. THE RECORDING DEVICE

While observations are made, one of the printed sheets of paper may be held under the photometer carriage by means of a flat table, movable at right angles to the photometer axis, or, as is done at the bureau, it may be wrapped on a cylinder (Fig. 1) whose axis is parallel to the photometer bar. By pressing a key located on the photometer carriage the observer prints on the paper a record dot for each setting as it is made, this being done by means of a printing electro-magnet suspended from the photometer carriage by a metal rod, the magnet being provided with two revolving spools carrying a carbon ribbon between the printing point and the sheet on the cylinder. The cylinder rests on the frame of the photometer bench and is turned on its axis by an electrically driven clock connected to it by means of a friction clutch which permits the cylinder to be turned by hand, if desired, without disturbing the clock. The clock and the printing magnet are operated in series by a battery of four storage cells, the magnet printing when the key is closed, the clock turning the cylinder a half mm when the key opens. The record dots are thus prevented from falling together, and by their relative positions they show the order in which the settings are made.

IV. THE NEW APPARATUS IN PRACTICAL USE

1. THE NECESSARY ADJUSTMENTS

The complete adjustment of the apparatus preliminary to making a series of measurements is simply as follows: The photometer carriage is set with the test side of the screen at 120 cm on the bar and the paper record sheet is wrapped on the cylinder and clamped, the index line having been previously placed immediately under the printing point and overlapped at its opposite ends. Then with a standard 16-cp lamp, burning at its correct voltage, in the socket at zero on the test side, and the comparison lamp burning at the proper voltage to match the standard in color, the rods connecting

the photometer and comparison lamp carriages are adjusted so as to throw the intensity balance anywhere within 6 cm either side of the index line, and the apparatus is ready for making measurements. The only adjustment requiring any degree of care is that of placing the index line at its proper distance of 120 cm from the lamp to be photometered.

2. RECORDING THE SETTINGS

Settings are now made on the standard lamp, and are recorded by a corresponding group of dots printed near the top of the sheet. The standard is then removed, the cylinder is turned by hand a distance sufficient to separate the group of dots just made from those to follow, and other standards, if desired, or lamps to be standardized are placed in turn in the socket and for each lamp the settings are recorded by a corresponding group of dots, as before. The record is thus continued down the length of the paper until all the lamps of the set are photometered.

A photographic reproduction of a completed record sheet, or chart, of a series of measurements actually made in this manner on a set of six standards and seventeen lamps compared with them is shown in Fig. 2, the lamps under measurement being part of a lot designated as "N. B. S." lamps. In the three columns of figures along the right margin are given the numbers of the lamps in the order they were photometered and recorded by the dots, together with their respective voltages and corresponding candlepower values. The cross mark under each group of dots indicates the average position of the individual dots of the group, this position being located usually by estimation. The candlepower values of this set were read from the linear scale *RS* whose correct position was determined in the manner described below. This will be referred to hereafter as the "record scale," being that scale which fits the adjustment of the apparatus and therefore of the record printed on the sheet.

3. READING THE RECORD

With the chart on a drawing board and by means of a T square with its edge parallel to the index line, preliminary relative candlepower values of the six standards were read from the standard scale and each reading was recorded by a mark on this scale as shown.

These readings, in the order of the record of the standards, were 16.15, 15.98, 16.09, 15.97, 16.40, and 16.08 cp, respectively, their mean being 16.11 cp. As the true mean value of these six standards happened to be 16.00 cp, it is evident that the linear record scale *RS* must lie above the standard scale and at such a distance from it that the 16.00-cp point on *RS* falls on the line *OO* which was drawn (by means of the T square) perpendicular to the standard scale at its 16.11-cp point.

The simplest method of determining the proper position for the record scale is to consider the line *OO* as a linear candlepower scale and draw *RS* intersecting it at the point where the reading is the mean true value of the standards; that is, at 16.00 cp for this particular case. No appreciable error is introduced by considering the line *OO* as a candlepower scale, because at this part of the rectangle the diagonal lines are so nearly parallel to one another that, for practical purposes, we may consider *OO* divided in the same proportion as the standard scale and the reading at the intersection of *OO* and the record scale will be the same on both.

With the record scale properly drawn, it becomes a simple matter (by means of the T square) to read off the correct values in candlepower directly and as rapidly as it is possible to set the edge of the T square over the centers of the various groups of record dots. It has been found by experience, however, that, instead of simply reading off the values in this manner, it is better practice to draw a vertical line through the intersection of each diagonal line with the record scale, a sufficient number of such vertical one-tenth candle lines being drawn to include all the groups of dots close together in value. In the case of any group that may be separated from the others, as, for instance, the one corresponding to lamp No. 892, the eighth in the set, it is better to draw a line directly from the center of the group to the scale. The advantage of having these lines on the chart is that the values can be more easily and more accurately read; and especially that the record is thus made more complete and permanent, permitting it to be checked in a few moments by a second reader.

The values of all lamps of the set, including the standards, are read off to the nearest 0.01 candle and written along the margin of the chart. If the linear record scale has been carefully drawn at

the proper distance from the standard scale, the mean of the standards, as read from it, will equal the true value to within 0.01 candle. Experience has shown that it is very seldom necessary to apply a correction to the final mean reading of the standards, showing that with ordinary care the record scale can be drawn with all the precision necessary. If, however, on account of inaccuracy in drawing the record scale, the mean reading of the standards should differ from the true value by a few hundredths of a candle, the difference is applied as a percentage correction to each reading in the set, being added or subtracted according as the mean of the standards reads too low or too high. Usually all the lamps of a set are so nearly equal in value as to permit a constant correction to each reading without introducing an error due to a lack of proportionality.

The true candlepower values of the individual standards used in the set given in Fig. 2 were, in the order recorded, 16.00, 15.90, 15.98, 15.82, 16.33, and 15.96, respectively. A comparison of these values with those read from the scale and written on the chart gives a fair indication of the accuracy of the settings made on the photometer.

4. USE OF SECTORED DISKS AND COLOR-ABSORBING SCREENS

When it becomes necessary to use rotating sectored disks to make the record of any lamp fall on the sheet, the final value of such lamp is determined by simply applying the disk factor in the proper manner to the value as read from the scale. For lamps of lower intensity than the standards, a balance at the desired place on the sheet may be obtained by placing the disk either on the comparison side when settings are made on the low-intensity lamps or on the test side when settings are made on the standards. In the former case the illumination of the photometer screen is not the same throughout the comparison, and there may be a possibility of error due to the Purkinje effect. In the latter case the illumination of the screen is the same throughout the comparison, but it can not be greater than that produced by the lamp of the lowest intensity, and may therefore be rather weak for accurate measurement. Although this latter method of measurement is preferable to the former when low-intensity lamps are measured,

yet it is inferior to a method that will be given when the sectional candlepower scale is described below.

For the most accurate measurement it is an advantage to work with a color match. When high-efficiency lamps, such as tantalums and tungstens, or carbon lamps operated at high efficiency, are compared with 4-wpc standards, a color match can be obtained by the use of carefully calibrated red or blue glass screens of the required density on the proper side of the photometer. With the comparison lamp set for 4-wpc color match with the standards, the blue glass is inserted on the comparison side, or the red glass on the test side when the high-efficiency lamps are photometered. Or, with the comparison lamp set for color match with the high-efficiency lamps, the blue glass is inserted on the test side when settings are made on the standards. On the other hand, when lamps of low efficiency are compared with 4-wpc standards a match in color is obtained by reversing this process. Whenever a color screen is used on the comparison side in either case, it should be replaced by a rotating sectored disk having approximately the same transmission coefficient as itself when the standards or other lamps of the set not requiring its use are photometered, in order to maintain the illumination of the photometer screen constant throughout the set of measurements. In any of these cases the transmission coefficients of the glass and of the disks used must be applied as factors in the proper manner to the candlepower values as read from the scale.

5. COMPARISON OF LAMPS WITH STANDARD ILLUMINATION OF THE SCREEN

It may be desirable, and is no doubt preferable, to compare low intensity lamps, such as 8 cp and 4 cp, with 16-cp standards with the illumination of the screen constant throughout the comparison and approximately 1 foot-candle as standard, rather than with lower illumination, as is necessary by the method described in the paragraph on the use of rotating sectored disks.

This may be accomplished by two different methods. By the first method all lamps compared are placed, in succession, at zero on the bar and settings are made without the use of sectored disks. With the apparatus so adjusted as to throw the balance for the 16-cp standards at 120 cm, all the settings are made with standard illumination of the screen, but they extend over a range

of at least 60 cm, falling at 120 cm on the bar for the standards and at 60 cm for the 4-cp lamps. This method is objectionable because it may be inconvenient, on account of some accessory part of the apparatus, to move the photometer carriage over so great range, and especially because, if the settings are to be automatically recorded, a continuous record sheet and scale of the required length (that is, of at least 70 cm) would possibly make the printed scale unreliable on account of changes in its length due to variations in atmospheric humidity, and besides the sheet would be very large and unwieldy for handling and filing purposes.

By the second method the settings are also made without the use of sectored disks and the apparatus is adjusted in the same manner as in the first method, but the lamps of low intensity are placed, in succession, at such points on the photometer bar that the settings fall at approximately 120 cm from zero for every lamp. For instance, an 8-cp lamp placed at 35 cm, or a 4-cp lamp at 60 cm, on the bar produces approximately the same illumination on the photometer screen at 120 cm as a 16-cp lamp placed at zero. With the lamps thus placed, in succession, all settings are made with standard illumination of the screen and near 120 cm on the bar, but the scale, as described above, will not serve for the correct reading of the candlepower of the lamps not placed at zero. Fortunately, however, the scale will adapt itself perfectly to the new conditions if it is divided into sections as shown in Fig. 3 and as described below.

V. THE NEW SCALE IN SECTIONAL FORM

1. METHOD OF CONSTRUCTION

In order to unaerstand clearly how the scale may be divided into sections so as to make it direct reading for lamps of such intensities as those mentioned above, when placed at the points indicated, let us suppose that the lamps are all placed at first, in succession, at zero, and that a record of the settings made on them is printed by dots on a continuous scale of the required length. The record of the 4 cp, the 8 cp, and the 16 cp lamps will then fall at 60 cm, 85 cm, and 120 cm, respectively, from zero. Now suppose that an index line is drawn across the scale at each of these three points and that the scale is divided into three sections, each containing an index line and the record dots corresponding to one of the lamps,

these being at or near the middle of the section. If the 8 cp and the 4 cp lamps are now moved to the points mentioned above; that is, to 35 cm and 60 cm, respectively, from zero on the bar, and if with each lamp the section on which the record of its settings ap-

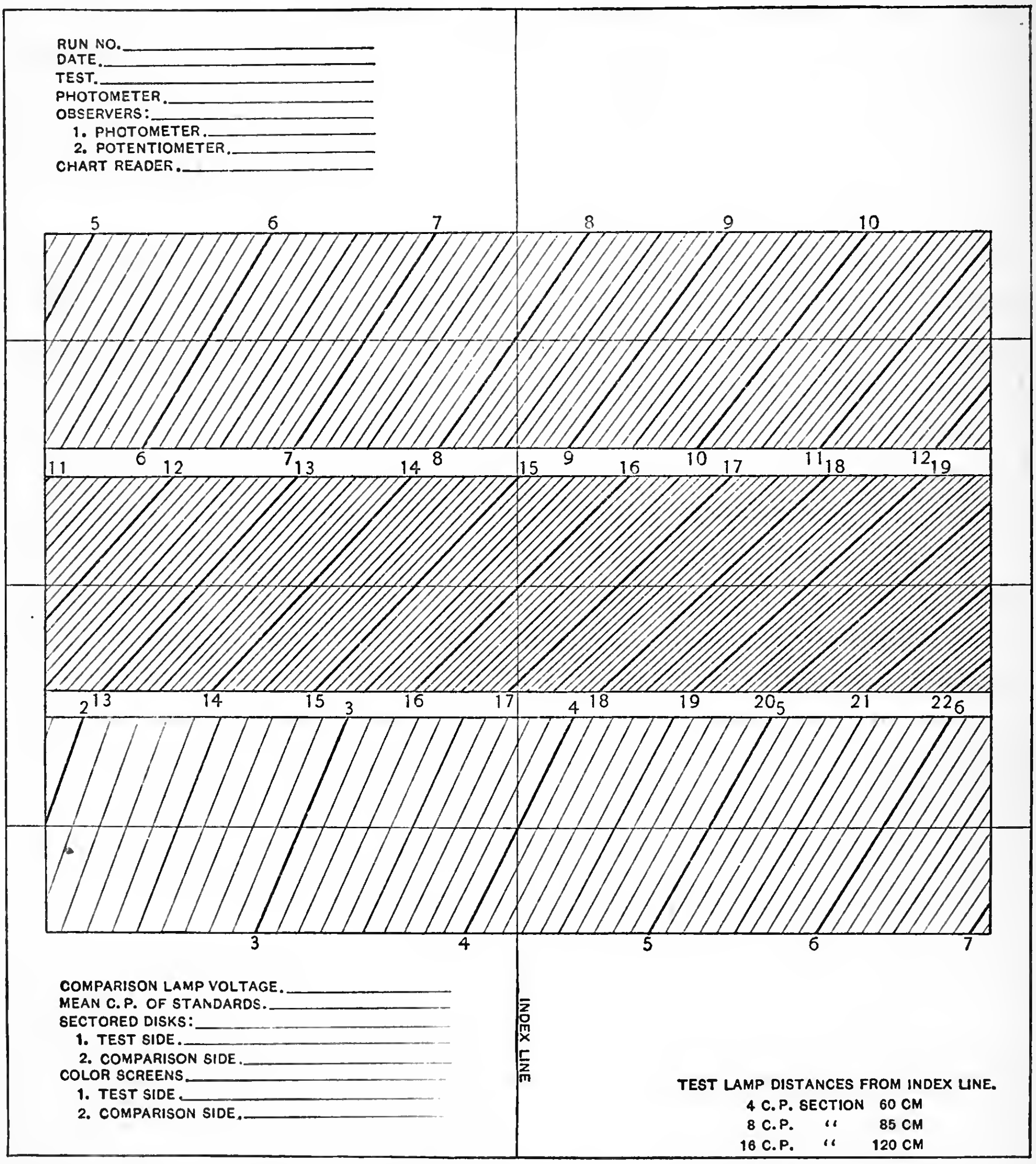


Fig. 3.—*Candlepower Scale in Sectional Form. Comparison Lamp Movable with Photometer Screen. Test Lamps may be Placed Successively at Different Points on the Photometer Bar.*

pears is moved forward by the same amount as the lamp, it is evident that the index lines of all three sections will fall together and that the record dots will occupy exactly the same relative positions with respect to one another as if they had been made with

the lamps located thus at three different points on the photometer bar. Hence, if these three sections (which will be referred to hereafter as "the 4-cp section," "the 8-cp section," and "the 16-cp, or standard, section," respectively) are placed, not superposed, but parallel and adjacent to one another as indicated in Fig. 3, with the index lines end to end as shown, thus forming a "sectional candlepower scale," which is then printed on the record sheet, the values of all the lamps of all three intensities may be read off directly in candlepower, each from its corresponding section of the scale.

As it will be seen, these three sections include all values continuously from 2 to 22 candles, there being a repetition in the region of 5 and 6 cp. This repetition was made purposely, in order to throw the 5-cp point nearer the middle of the scale than it falls on the 8-cp section, this being desirable because the bureau at present has more or less standardizing to do in this region of intensity and the record dots are thus at a point on the scale where a change in the length of the scale would be less liable to produce an appreciable error in the reading. The 16-cp, or standard, section was drawn in the middle portion of the sectional scale in order that it may lie between the two parts of the record of the settings on the standards which usually appear at the top and bottom of the sheet; and also for the reason that the greater number of lamps intended for standards are of such intensities as are read from this section. With a centimeter space allowed between adjoining sections, the over-all dimensions of this sectional scale are 35 cm in length by 26 cm in width. The individual sections are 8 cm in width, this being found by experience with the scale shown in Fig. 2 to be ample for all practical purposes. This scale is now being engraved and printed for the bureau on record sheets 40 cm by 37.5 cm, the latter dimension being in the direction of the length of the scale, and it will be used whenever lamps of low intensity are to be photometered.

2. ADAPTATION TO THE PRINTED RECORD

It is evident that the record scale should appear on every section and at exactly the same distance from the standard scale on each section, as would be the case were the sections together in one continuous diagonal scale. It is first located on the standard

section, in the usual manner, and then drawn at the same relative position on the other two sections. This drawing is quickly and accurately done by means of (what we shall call) a "triple right-angle sliding index" (shown in Fig. 6), having the three corresponding edges, which are perpendicular to the edge of the T square, exactly 9 cm apart, so that when the middle edge is placed on the standard scale of the 16-cp section, the other two edges will fall exactly on the standard scale lines, respectively, of the other two sections. Hence, when it is correctly placed to draw the record scale on the 16-cp section, it is at the same time in correct position to draw the corresponding line on each of the other two sections, and all three lines may be drawn without moving the index.

Although corresponding to each section of the scale, there is a definite distance (which for the sake of brevity will be referred to as its "lamp distance") at which the lamp must be placed with respect to the common index line on the sectional scale, if its value is to be read off directly, nevertheless, the scale as a whole readily adapts itself to any other lamp distance than those specified. For example, suppose that a lamp is for some reason at a distance of 65 cm from the common index line. Its value is read from the 4-cp section, but at a point 5 cm higher on the scale than is represented by the record. This is for the reason that had the lamp been placed at the exact lamp distance for the 4-cp section (that is, at 60 cm) instead of at 65 cm, the record would have fallen just 5 cm higher on the scale, in which case the reading would have been direct. Supposing, for a second example, that the lamp distance is 75 cm; the value is then read from the 8-cp section (whose lamp distance is 85 cm) at a point 10 cm lower on the scale than is represented by the record. These higher or lower readings are taken by means of a centimeter scale graduated on the edges of the triple index, as shown in Fig. 6.

Another condition, which is equivalent to placing every lamp of a set at an incorrect distance in the same direction by the same amount, exists when the common index line on the record sheet is not exactly at 120 cm from zero of the scale when the settings are recorded. The values may be read off correctly in the same manner as above by allowing the correction in distance when reading the scale. For instance, if the common index line is set at

121 cm from zero, then all the record dots are 1 cm too low and hence each reading, to make it direct in candlepower, should be taken just 1 cm higher on the scale.

3. USE IN COMPARING FLAME AND ELECTRIC INCANDESCENT STANDARDS

A sectional scale, such as just described, is useful in the comparison of a flame standard with a standard electric incandescent lamp, in which it is not only inconvenient but it is also undesirable to place the lamps compared, successively, at the same point, because the flame standard should be disturbed as little as possible during the comparison. In standardizing, or investigating, a 10-cp pentane lamp, for example, it is most convenient to have it located on a table at the end of the photometer bench with the flame in line with the photometer axis and at a distance of about 30 cm from the zero point; and to have the incandescent lamp placed, in turn, at some point on the photometer bar, the light from the pentane lamp being screened off while the incandescent lamp is measured. In this way the flame standard need not be disturbed at any time during the comparison.

As it is desirable to photometer the pentane lamp at the standard distance of 100 cm, as adopted by the London Gas Referees, the scale to be used in such comparisons should be constructed on the basis that 10 cp falls at exactly 100 cm from zero, whereas on the scale shown in Figs. 2 and 3 it falls at approximately 95 cm from zero. The distance between the index lines on the undivided scale should, of course, be the same as the distance between the points at which the lamps compared are located when each, in its turn, is photometered.

4. AS CONSTRUCTED FOR USE WITH AN INTEGRATING PHOTOMETER

In the case of some instruments, such as the Matthews integrating photometer and the Ulbricht sphere, the test lamp and the photometer screen remain stationary, the settings being made by varying the distance of the comparison lamp. The sectional scale for this arrangement is constructed on the same principle as the one already described, but the equation used in computing the candlepower points is $x = 4a\sqrt{\frac{1}{X}}$ since here the candlepower X, of

the test lamp varies inversely as the square of the distance x, of the comparison lamp. In this case only one section of the scale is useful at any given position of the cylinder on the photometer bench. For lamps higher or lower in candlepower the cylinder is moved the proper direction to a new position bringing another section of the scale into use. The scale is properly located by first making an approximate setting on a standard then adjusting the position of the cylinder so that the middle of the record sheet comes under the printing point of the magnet (suspended from the comparison-lamp carriage) or, if possible, so that the common index line falls at a distance from the screen exactly equal to the lamp distance corresponding to one of the sections. But should the distance of the index line differ from the lamp distance by a known amount, the allowance is made as described above when reading the scale.

VI. THE SCALE ADAPTED TO PHOTOMETERS HAVING BOTH LAMPS STATIONARY

1. THEORETICAL DISCUSSION

The candlepower scale in the forms thus far described is adaptable only to such photometers as have either the test lamp or the comparison lamp at a constant distance from the photometer screen. In these forms all the candlepower lines are theoretically straight and join corresponding points of division in two linear scales forming the upper and lower sides, respectively, of the rectangle. It will now be shown that the same method may be used in constructing the scale in a form adaptable to photometers having both lamps stationary.

Let C and T be the values of the comparison lamp and the test lamp, respectively, D, the constant distance between them, and x the distance of the screen from the test lamp. Then for photometric balance

$$\frac{T}{C}=\frac{x^2}{(D-x)^2} \tag{1}$$

from which

$$x=\frac{D}{1+\sqrt{\frac{C}{T}}} \tag{2}$$

In order to plot the locus of the balance point of the photometer screen corresponding to a given value of T, while the value of the comparison lamp is varied continuously, let the values of x be distances measured on the X-axis of coordinates, and let distances to be measured on the Y-axis be represented by

$$y = \frac{D}{1 + \sqrt{\frac{T_o}{C}}} \tag{3}$$

where C has the same value as in equation (2), while T_o is a particular value of T in the same equation; that is, the Y-axis is regarded as a linear candlepower scale for the comparison lamp when the test lamp is held constant at the value T_o.

In order that the locus of the point x, y may be plotted without a knowledge of the value of the comparison lamp, let C be eliminated by equating its value in equations (2) and (3), and thus obtain either

$$x = \frac{D(D-y)\sqrt{T}}{D\sqrt{T} + (\sqrt{T_o} - \sqrt{T})y} \tag{4}$$

or

$$y = \frac{D(D-x)\sqrt{T}}{D\sqrt{T} + (\sqrt{T_o} - \sqrt{T})x} \tag{5}$$

Now, in either equation $x = D$ when $y = o$, and vice versa, showing that the locus of the point intersects both axes at a distance D from the origin.

In the special case when $T = T_o$, both equations reduce to $x + y = D$, meaning that the locus for this case is a straight line. A further discussion of this case will be taken up below.

Since x and y are interchangeable in equations (4) and (5), it follows that the locus is symmetrical with respect to a line OE bisecting the right angle XOY between the axes. (See Fig. 4.)

If the coordinates are transformed so as to refer to a pair of axes parallel to the original ones by the substitution of $x' + a$, and $y' + a$ for x and y, respectively, in either (4) or (5), a being assumed equal to $\frac{D\sqrt{T}}{\sqrt{T} - \sqrt{T_o}}$, the equation reduces to

$$x'y' = a^2\sqrt{\frac{T_o}{T}} \tag{6}$$

Since the second member of equation (6) is a constant for any given values of T and T_0, the locus of the point x', y' is an equilateral hyperbola asymptotic to a pair of axes parallel to the XY axes, and at a distance of a from them, a of course being a different constant for each different curve of the system.

In the limiting cases when either T or T_0 is separately equal to 0 or ∞, the locus in each case degenerates into the axes of coordinates. When $T = T_0$ the locus consists of two parallel straight

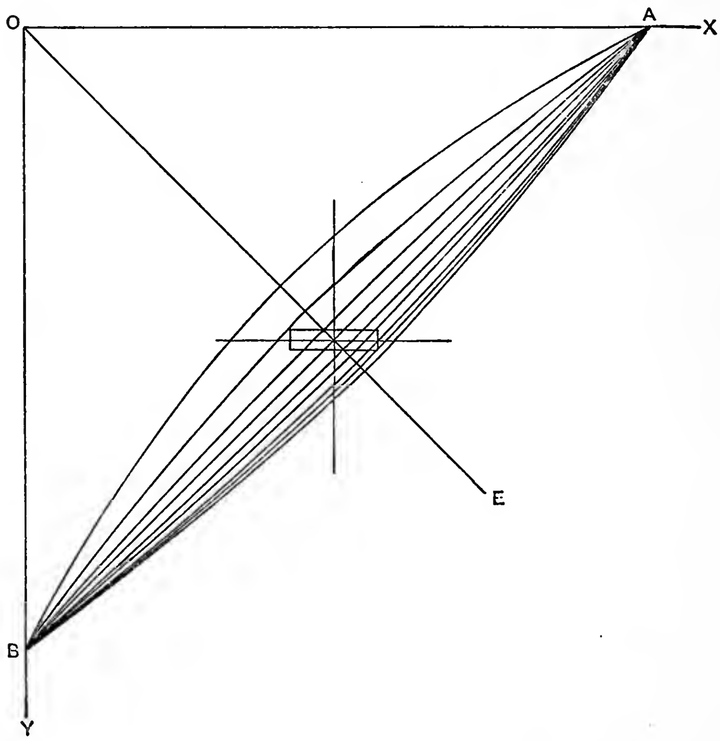


Fig. 4.—*System of Candlepower Lines Represented by Equation (4) or (5)*

lines at an infinite distance, one on either side of the origin, and perpendicular to a line bisecting the angle between the axes. The former cases are of interest only from a mathematical standpoint; the latter, however, is important because under this condition one branch of the locus, referred to above, becomes (as will be seen later) the reference line of the system when the lines are drawn as actually used in the scale and the coordinates are referred to the XY axes as in equations (4) and (5).

Hence, if by means of either of the original equations (4) or (5), the loci of the point x, y for successive values of T such as met with in practice (T_0 being kept constant) be plotted and drawn,

there is obtained a system of lines such as shown in Fig. 4, all the lines being curved except the one corresponding to $T=T_0$, each line of course being one of a pair of which the locus consists. In this drawing y is considered positive when measured downward from the origin, in order to have the candlepower lines slope consistently with those of the other forms of the scale hereinbefore illustrated. The particular curves shown correspond to values of T beginning with 4 candlepower for the one nearest the origin and increasing by steps of 4 candlepower for the series, the value of T_0 being 16 candlepower.

It is evident that any section cut out from the system of lines by a rectangle parallel to the X-axes, as shown in Fig. 4, is a candlepower scale similar to those already described, but adapted to photometers having both lamps stationary at a distance D apart.

Although all the candlepower lines, with the exception of the one corresponding to $T=T_0$, are curved theoretically, it is found that within the limits of a scale such as shown in Fig. 5, and indicated in correct proportion in Fig. 4, the straight line of the system lying, as it should, at the middle of the scale, the maximum deviation from a straight line, as will be shown later, is less than the accuracy within which it is possible to plot and draw the candlepower lines at their proper positions in the scale. Hence, this form of the scale may be constructed of straight lines in precisely the same manner as those already described.

2. CONSTRUCTION OF THE SCALE

When using 16 candlepower standards, it is convenient to have the 16 candlepower line at the middle of the scale, and since the straight line of the system can be made to correspond to any value chosen for T_0, let this value be 16 candlepower. Then for a photometer bench such as used in commercial work at the bureau having $D=250$ cm, equation (4) becomes

$$x=\frac{250\,(250-y)\sqrt{T}}{250\sqrt{T}+(4-\sqrt{T})y} \tag{7}$$

by means of which the scale shown in Fig. 5 was computed, the dimensions of the rectangle being 8 cm by 35 cm. The upper and

lower sides of the rectangle were divided into linear candlepower scales, just as in constructing the other forms, using here values of x determined by means of the above equation, and the corresponding candlepower points in the two scales were joined by *straight* lines. For the upper linear scale y had the value 121 cm, and for the lower 129 cm, these distances having been chosen so as to throw the middle point of the 16 candlepower line at 125 cm, the point midway between the lamps.

Theoretically, the maximum departure of the 13-cp line from a straight line is 0.049 mm; of the 11-cp line, 0.08 mm; and of the portion of the 9-cp line appearing, 0.03 mm. The system of curves being symmetrical with respect to the middle cp line ($T=T_0$), the same amount of curvature is to be found at the other end of

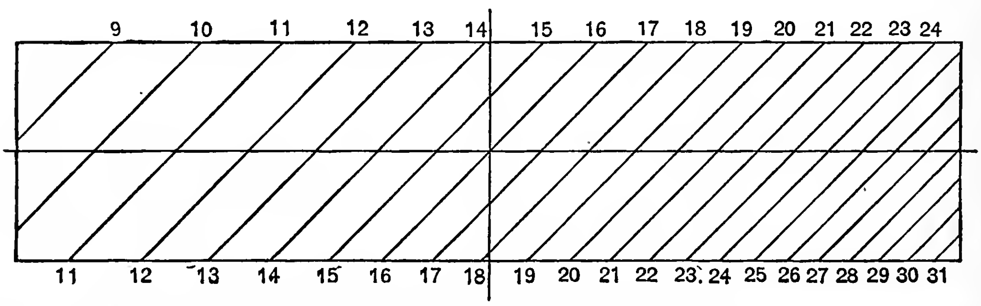


Fig. 5.—*Candlepower Scale Adapted to Photometers Having both Test and Comparison Lamps Stationary*

the scale. Therefore, within the limits of the lines appearing in this scale, the maximum departure from a straight line in any case is so small that it is impossible, in practice, to draw a straight line so narrow that it will not cover every point of the curve.

Since the curvature of the lines is greater the farther they lie from the line $T=T_0$, it must be determined where the curvature becomes appreciable if the scale is to be extended a considerable distance either way from $T=T_0$, and from that part of the scale to the end a sufficient number of points should be plotted for each curve to insure the required accuracy in drawing.

3. RELATIVE SENSIBILITY

Merely a glance at the two forms of the scale as illustrated in Figs. 2 and 5, respectively (which are drawn with the 16-cp line at nearly the same distance from zero; that is, at 120 cm and

125 cm, respectively), is sufficient to show that a photometer setting can be read with a greater degree of accuracy by means of the former than by means of the latter. This degree of accuracy may be called the "sensibility" of the scale and may be expressed mathematically as the derivative of x with respect to T, which, as will be seen, is a variable depending upon the value of T. Likewise the ratio of these derivatives in the two forms of the scale is a variable depending upon the value of T; but over the range of the cp lines appearing in these two forms, the average value of this ratio is approximately two, if we assume that the cp point corresponding to $T=T_0$ falls at the middle of the scale and at exactly the same distance from zero in the two cases. This may be shown in the following manner

The equation for a linear scale in Fig. 2 is

$$x=a\sqrt{\frac{T}{C_1}} \tag{8}$$

and for one in Fig. 5,

$$x=\frac{D}{1+\sqrt{\frac{C}{T}}} \tag{9}$$

where in equation (8) a is the constant distance of the comparison lamp C_1 from the screen. Now let the cp point corresponding to $T=T_0$ fall at a distance $\frac{D}{2}$ from zero in each scale. In order that this may occur, the illumination on the comparison side of the screen must be the same in both cases when the balance is at this point; that is

$$\frac{C_1}{a^2} \text{ must equal } \frac{4C}{D^2}$$

As C and D are constants, C_1 may have any value we choose so long as a has the proper corresponding value. Let us then assume $C_1=C$ and therefore $a=\frac{D}{2}$. Hence equation (8) becomes

$$x=\frac{D}{2}\sqrt{\frac{T}{C}}. \tag{10}$$

Now, according to the above definition, the sensibilities of the two forms of the scale as derived from equations (9) and (10) are

$$S_1 \equiv \frac{dx}{dT} = \frac{DC^{\frac{1}{2}}T^{-\frac{3}{2}}}{2(1+C^{\frac{1}{2}}T^{-\frac{1}{2}})^2}$$

and $$S_2 \equiv \frac{dx}{dT} = \frac{DC^{-\frac{1}{2}}T^{-\frac{1}{2}}}{4}, \text{ respectively,}$$

and hence their ratio is

$$R_1 \equiv \frac{S_2}{S_1} = \frac{(T^{\frac{1}{2}}+C^{\frac{1}{2}})^2}{2C}$$

which, as stated above, is a variable depending upon the value of T. At the middle of the scales where $T = T_0 = 16$, $R = 2.00$; while at the 9-cp and the 25-cp lines, $R = 1.53$ and 2.53, respectively, or an average of 2.03.

In the same manner it may be shown that the sensibility of the form of the scale adapted to photometers having both the test lamp and the screen stationary, the comparison lamp alone being movable, is

$$S_3 \equiv \frac{dx}{dT} = -\frac{DC^{\frac{1}{2}}T^{-\frac{3}{2}}}{4}$$

(derived from $x = a\sqrt{\frac{C}{T}} = \frac{D}{2}\sqrt{\frac{C}{T}}$, the equation of the scale) and that

its ratio to the sensibility of the form adapted to photometers having both lamps stationary is

$$R_2 \equiv \frac{S_3}{S_1} = -\frac{(1+C^{\frac{1}{2}}T^{-\frac{1}{2}})^2}{2}$$

(the negative sign indicating that the scales run in opposite directions.)

In this case when $T = T_0 = 16$, $R = -2.00$; and at $T = 9$ cp and $T = 25$ cp, $R = -2.72$ and -1.62, or an average of -2.17.

Hence it may be stated that the form of the scale adapted to photometers having both lamps stationary is only about one-half as sensitive as either of the other two forms described.

It must not be concluded from this statement that the photometer itself is less sensitive in one case than in the others; for with the proper means of moving the photometer head back and forth by very small amounts and setting it at exactly the point desired, it should be possible, theoretically, to make settings with the same percentage accuracy, regardless of the arrangement of the photometer. The above conclusion means simply that with a setting once made the accuracy with which its position on the scale can be read depends upon the arrangement of the photometer; and hence for the most accurate work the best results are to be obtained by using a photometer having one lamp stationary with respect to the screen.

VII. SPECIAL USES OF ALL FORMS OF THE SCALE

1. READING OF RECORD PRINTED ON BLANK PAPER

Should the means of taking exact copies be not at hand it would of course, be very laborious to draw a separate scale for each set of measurements. It is a simple matter, however, by means of but one copy of the required form of the scale (whether it be continuous or sectional) carefully drawn on a separate sheet of paper or other material to read off directly in candlepower the values recorded by dots on a sheet of paper that is otherwise perfectly blank, with the exception of an index line, which may be drawn either before the paper is placed on the cylinder or at any time before it is removed by means of a pencil held at the proper point while the cylinder is given one complete turn on its axis.

The values may be read off by several different methods. By one method the record sheet and the scale sheet are clamped side by side on a drawing board, as shown in Fig. 6, with the index line of the one in line with that of the other, and readings are taken by means of a double T square provided with a fine wire stretched parallel to its edge and also with a triple right-angle sliding index (such as shown in the figure and described above) whose three straight edges are perpendicular to the wire and therefore to the edge of the T square. With the wire kept parallel to the index line and placed successively over the center of each group of record dots relative candlepower values of the standards are read from the standard scale on the 16-cp section. The wire is then set at

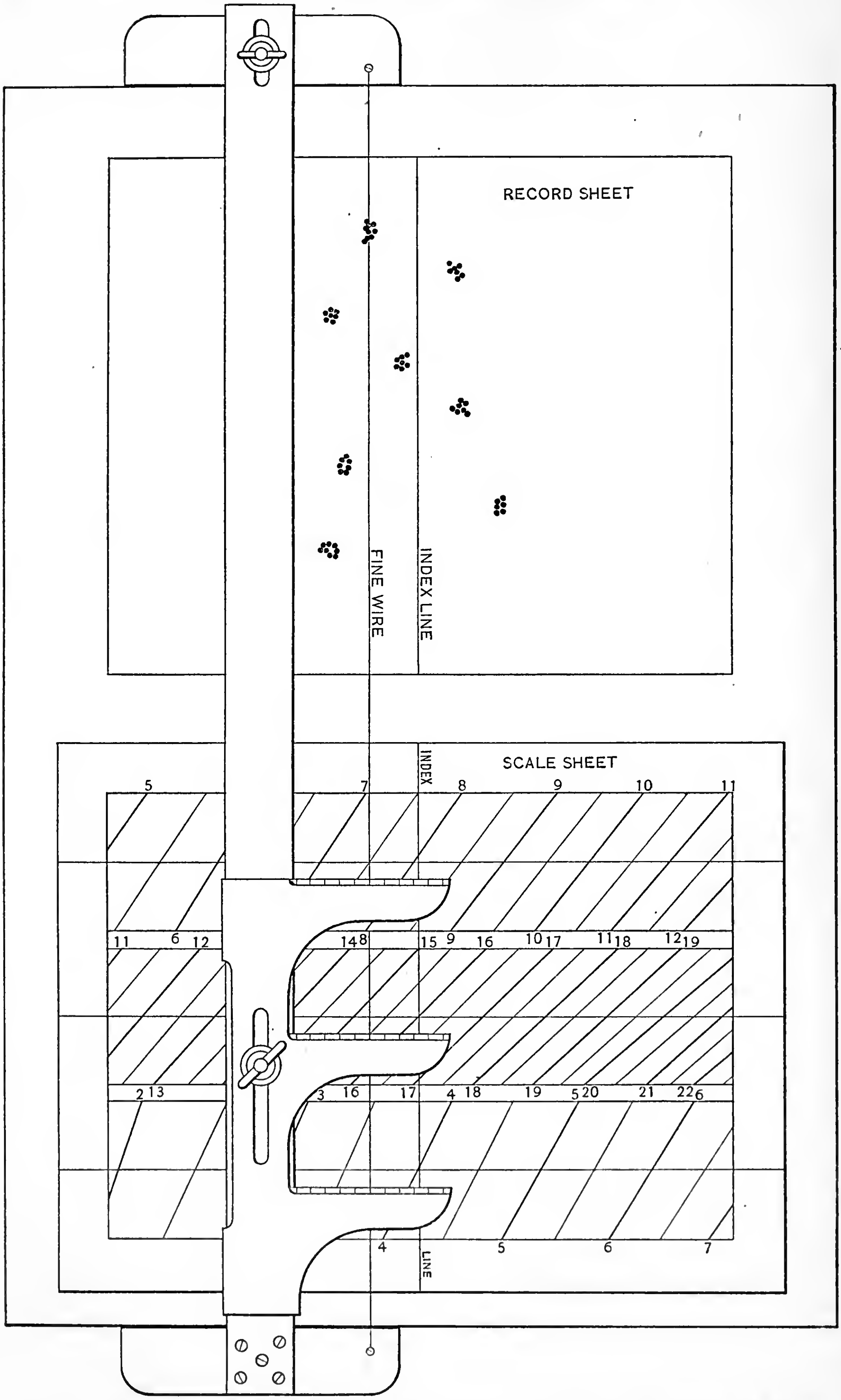


Fig. 6.—*Showing Method of Reading, by Means of the Scale, a Record Printed on a Separate Sheet.*

the mean of these readings and the triple index is moved until the intersection of its middle edge with the wire falls immediately over the exact value of the mean of the standards, at which point it is permanently clamped to the T square. Each edge of the sliding index now corresponds to a linear record scale, and the values of the test lamps may be read from the proper scale and section as fast as it is possible to set the wire over the center of gravity of each successive group of dots. The accuracy of the adjustment of the sliding index is indicated by the accuracy with which the mean of the standards, as read, equals the true value, the index being reset before the test lamps are read in case a correction is necessary. Vertical one-tenth candlepower lines may be drawn on the record sheet, if desired, by means of the edge of the T square, and the readings are then taken from the record sheet itself.

Another method of using the diagonal scale in determining the values from the record printed on blank paper is to have the scale ruled on glass or other transparent substance that will, under ordinary conditions, remain constant in dimensions. With this placed on the record sheet with the index lines superposed the values may be read off by means of the T square and sliding index in the manner described above.

Still another method is to have the record dots printed on transparent paper which may be placed over the scale with the index lines superposed, and the values may be read off directly by means of the T square and sliding index, or all the vertical one-tenth candle lines may be drawn as in the first method and the readings taken from the record sheet. The same thing may be accomplished with ordinary paper, provided the record sheet is placed over the scale sheet on a ground-glass plate illuminated from below while readings are taken or the lines are drawn on the paper. It is an advantage, whenever possible, to actually draw on the record sheet the lines mentioned, because the record is more complete and it is more easily checked by a second reader.

2. CANDLEPOWER READ DIRECTLY WITHOUT A PRINTED RECORD

Although for the sake of obtaining a permanent record, the settings are usually recorded by dots on the sheet, yet it is not at all necessary to do so. By means of a cross-hair reading index

suspended from the photometer carriage the relative values of the standards may be read directly from the standard scale of the proper section which is brought immediately under the cross hair by turning the cylinder. The cross hair is then set at the mean reading of all the standards and the cylinder is turned until the reading on the scale corresponding exactly to the true value of the mean of the standards falls immediately under the cross hair. The cylinder is then clamped to prevent it from turning on its axis, and as the clock is not required, the clutch connecting it with the cylinder is released. Now as the photometer carriage is moved back and forth, the path of the intersection of the cross-hair index corresponds to the record scale on the standard section. In order to make it possible to take readings from the other two sections of the scale also, if desired, the cylinder is clamped, not directly to the frame on which it rests, but to a circular disk mounted concentrically on the same axis with itself, at the end opposite the clock, and the disk in turn is held rigid with the frame by means of a stationary pawl which engages one of three notches in its circumference, each notch corresponding to the record scale on one of the sections. This notched disk and pawl now take the part of the triple sliding index used in drawing the record scale in each section as described above. If, for instance, the standards are read from the middle section of the scale, the pawl is made to engage the middle notch when the cylinder is set and clamped to the disk and either of the other two sections is in correct position to take readings when the cylinder has been turned the proper direction and the pawl has engaged the notch corresponding to the section desired.

With the scale used in this manner, it becomes a valuable adjunct to a commercial photometer, for which purpose it may be engraved directly on the metal cylinder itself. It has all the advantages of the ordinary commercial scale and has the further advantage of quick and accurate adjustment without a tedious adjustment of the comparison lamp, which is arbitrarily set at the beginning of a set of measurements and need not be changed thereafter either in voltage or in distance.

VIII. SPECIAL METHODS OF RECORDING SETTINGS

1. WHEN RECORDING LONG OR SHORT SERIES OF MEASUREMENTS

Although a single record sheet is usually sufficient for a series of measurements on a set of lamps, yet the record may be continued on a second sheet, if necessary, without repeating the standards used at the beginning, for it is a simple matter to put the second sheet at exactly the same place on the cylinder and to draw the record scale in the same relative position on the diagonal scales on the two sheets.

On the other hand, two or more short series of measurements, made by as many different observers, may be recorded, one after another, on the same sheet. Indeed, all the different observers may make measurements on the same lamp, one after another, before proceeding to the next lamp in the set, and the different observers' values may be read off separately and correctly. Of course the position of the linear record scale may be, and most probably will be, different for each observer's record, even though no change be made in the adjustment of the apparatus, this being due to the personal equation and to a difference in the eyes of the observers. This method of measuring and recording is sometimes useful in comparing a single very special lamp with the standards.

2. WHEN STANDARDIZING AT A SPECIFIED CANDLEPOWER

In much of the bureau's work it is necessary to standardize lamps at a specified value, as for instance, 16 cp. This scale has proved itself valuable in such cases in the way of saving time, because it can be very accurately determined in advance where the reading for the required value should fall. It is customary to read three of the six standards at the beginning of a set of measurements and include the other three at the end, in order to detect any possible, accidental changes in the adjustment of the apparatus and also in order that any continuous change in the comparison lamp, or in the eye of the observer, may be eliminated in the mean of the standards and in that of the test lamps. After the first three standards are recorded their relative values are read from the standard scale and the mean determined. In making these readings the printing point is set over the center of the group to be read, the scale being brought under the point by turning the cylinder. If the value required for the test lamp

is of the same nominal candlepower as the standards, then the exact difference in candlepower between the value required and the true mean value of the three standards is applied algebraically to the mean reading and the resultant value is marked at the proper point on the standard scale of the 16-cp section. For example, if the mean reading of the three standards is 16.19 cp on the standard scale, while the true value is 16.07 cp, and the value required for the test lamp is 16.00 cp, then the reading on the standard scale corresponding to this required value is 0.07 of a candle less than 16.19 cp, or 16.12 cp.

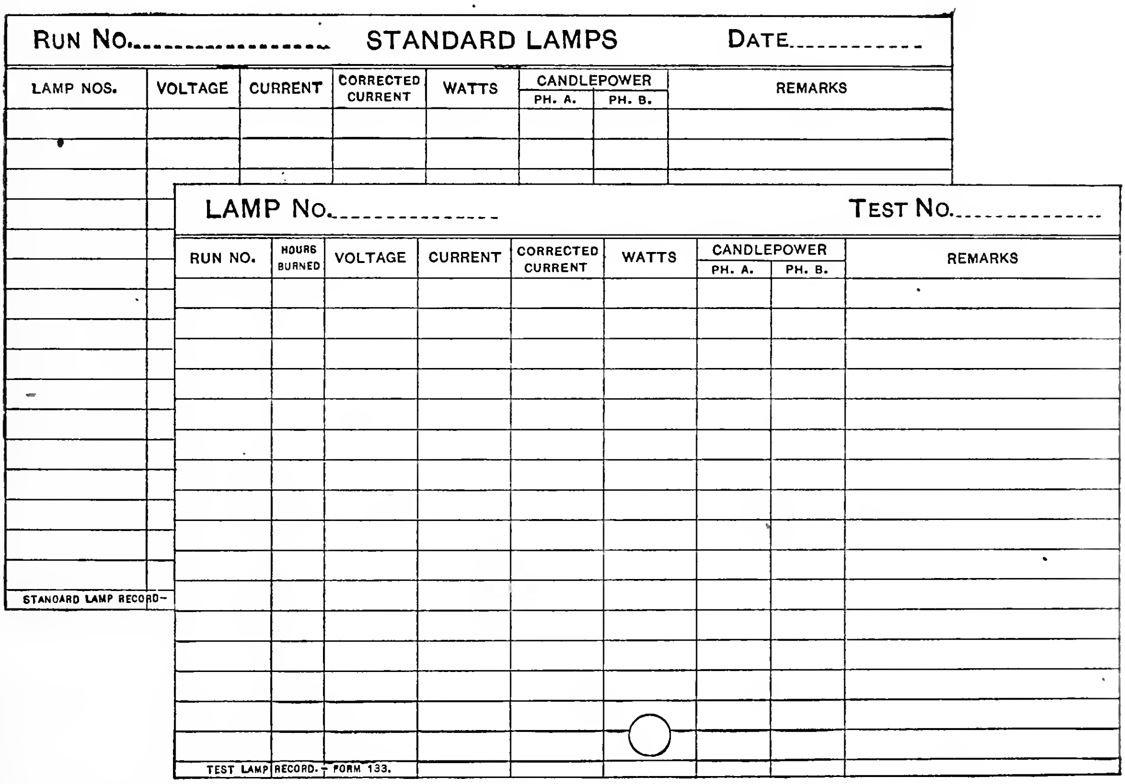
A reference line can be very readily drawn through the mark just made at 16.12 cp on the scale by the method described above, i. e., by holding the point of a pencil on the mark while the cylinder is given one complete turn. This line will, of course, be drawn upon the basis of the readings on but three of the standards and may need a small correction when the readings on the other three standards are considered, but for practical purposes it is sufficiently near to the position where the record dots for 16.00 cp should fall. This same line serves for the standardization of a lamp to be any multiple of 16.00 cp, if the proper sectored disk is used on the test side during the measurement, or any fraction of 16.00 cp if the lamp is placed at the proper point on the photometer bar, in which case the reading will, of course, be taken from the corresponding section of the scale.

The cylinder is then turned back to where the record was interrupted after the three standards were recorded, and with the printing point placed over the reference line just determined and the test lamp in the socket the voltage on the lamp is raised by the assistant at the potentiometer until the observer at the photometer is satisfied that there is a photometric balance. The voltage is then held constant while the observer proceeds to make settings and corresponding record dots. Although it is almost impossible to make the mean of the group of dots fall exactly on the reference line, yet with a little practice the observer is, in this way, able to determine very closely the proper voltage for a given candlepower value. The relation between candlepower and voltage is so well known for short ranges that any small correction that it may be necessary to apply to the observed value, to give the required candlepower, can be very accurately made by a corresponding calculated change in the voltage.

IX. MEASUREMENT AND RECORD OF ELECTRICAL QUANTITIES

1. THE ELECTRICAL MEASUREMENTS

In all the precision photometric work of the bureau the current is supplied by a storage battery carrying no other load, and both current and voltage are measured by means of a potentiometer. While the photometer settings are made the assistant at the potentiometer controls the voltage on the lamps on the two sides of the photometer and measures the current in the lamp under observation.


RUN NO.................. STANDARD LAMPS DATE............

LAMP NOS.	VOLTAGE	CURRENT	CORRECTED CURRENT	WATTS	CANDLEPOWER		REMARKS
					PH. A.	PH. B.	

STANDARD LAMP RECORD—

LAMP NO............... TEST NO...............

RUN NO.	HOURS BURNED	VOLTAGE	CURRENT	CORRECTED CURRENT	WATTS	CANDLEPOWER		REMARKS
						PH. A.	PH. B.	

TEST LAMP RECORD.—FORM 133.

Fig. 7.—*Cards for Recording Voltage and Current and for Complete Summary*

2. RECORD OF ELECTRICAL MEASUREMENTS AND FINAL SUMMARY

In addition to making the electrical measurements, the assistant records on a filing card the current in each lamp as it is photometered, there being a separate card for each lamp (see Fig. 7) except that the record of the six standards is kept together on a single card for the sake of convenience in determining their mean and also of keeping a more condensed record of the standards. The numbers of the lamps and their corresponding voltages are

entered on the cards previous to making the run, and all the test-lamp cards are linked together in the order in which the lamps are to be photometered, so that after the record dots are printed no error will be introduced in recording the numbers of the lamps in the proper order opposite the groups of dots corresponding to them. These numbers might be written on the sheet before the lamps are photometered, thus providing the observer with a check on the assistant who reads the numbers of the lamps as he places them in the socket, but it is not always possible to make the record dots fall opposite the numbers desired.

The test lamp card contains a column for the number of hours the lamp has been burned, if it required seasoning, and both the standard and the test lamp cards contain a column for corrected current and two columns for candlepower. The column for corrected current is required because when the voltage is measured by means of a potentiometer part of the current passes through the accompanying volt box, and hence a correction must be applied to the current as measured. Two columns are required for candlepower because measurements are now made by two observers at the same time on the same lamp, this being done by means of a double photometer[2] consisting of two standard photometers (referred to as photometer A and photometer B, respectively), such as already described in this paper, placed end to end, the test lamp being located at the middle where the two photometers meet. This arrangement permits double the number of measurements in a given time, requires the standards to be burned only one-half as long for a given number of measurements, and especially makes the settings of the one observer a valuable check upon those of the other.

By the data given on the candlepower record sheet (Fig. 3) and on the filing cards (Fig. 7) there is formed a complete historical record of all the work done on a lamp while under observation, and it is in such simple form as to be very conveniently filed, a complete cross reference being provided by a serial number appearing on every record sheet and card connected with any given set of measurements.

[2] Described in the paper referred to in the introduction.

X. CONCLUSION

This combination candlepower scale and recording device has been in use in the Bureau of Standards almost daily during the past year and has proved itself entirely satisfactory. It has eliminated the computations which formerly occupied the entire time of at least one computer, and sometimes that of two. The results of a half day's work by two observers on the double photometer can be read from the scale and written on the record sheets in a few minutes, and the final record can be checked entirely by inspection. There are no tedious adjustments required and the photometer settings are recorded rapidly and accurately. The eye is not unnecessarily fatigued by alternate reading of the photometer screen and scale, and the observer is almost entirely freed from prejudice in making the settings. The record is simple, permanent, and complete, and is easily read, checked, and filed for future reference.

WASHINGTON, June 6, 1910.